TECHNICAL AIDS

by
Lloyd S. Nelson

The Mean Square Successive Difference Test

Successive differences are simply the second observation minus the first observation, the third minus the second, and so on. Quality engineers will recognize these as moving (or running) ranges of two. When each difference is squared and the average calculated, the result is termed the mean square successive difference. Finally, when this result is divided by the sample variance, a statistic is obtained that can be useful in process capability studies. Call the statistic M. In symbols, we define M in the following manner.

$$M = \frac{\frac{1}{n-1}\sum_{i=1}^{n-1}(x_{i+1}-x_i)^2}{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2} = \frac{\sum_{i=1}^{n-1}(x_{i+1}-x_i)^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2} \qquad (1)$$

As can be seen from equation (1), the factor $(n-1)$ cancels and M can be obtained by dividing the sum of the squares of the successive differences by the sum of squares of the deviations of the observations from their mean.

The statistic M is used to detect nonrandomness in a series of observations. When data come from a process that is in control, the average value of M is 2. Rapid fluctuations (a saw-tooth pattern) will yield a larger value of M. Slow oscillation will cause M to be small. Newly computed values for the 0.10, 0.05, and 0.01 levels of significance were generated using the series approximation given as equation (9) in Hart [2] with an algorithm given by Woods and Posten [3] for the incomplete beta function ratio. It is assumed that the observations come from a normal distribution. This can be considered an extension of Table 11.5 in Bennett and Franklin [1]. To conserve space, only the lower percentage points P_L are given in Table 1 (for testing whether M is too small). Upper percentage points P_U, for testing whether M is too large, are easily obtained from $P_U = 4 - P_L$.

In an example taken from [1], to test for serial correlation in the following plant yields, the M statistic is appropriate. Data for 26 consecutive weeks are 81.02, 80.08, 80.05, 79.70, 79.13, 77.09, 80.09, 79.40, 80.56, 80.97, 80.17, 81.35, 79.64, 80.82, 81.26, 80.75, 80.74, 81.59, 80.14, 80.75, 81.01, 79.09, 78.73, 78.45, 79.56, 79.80. The sum of squares of the 25 successive differences is 31.7348; the conventional sum of squares for the 26 observations is 26.4006. Thus $M = 31.7348/26.4006 = 1.20$. Reference to Table 1 shows that for a sample of size $N = 26$ significance has been reached at between the 0.05 and 0.01 levels. Conclusion: there is a nonrandom fluctuation in the form of a slow oscillation in these data.

KEY WORDS: Mean Square Successive Difference Test, Serial Correlation, Trends

References

1. Bennett, C. A. and Franklin, N. L., *Statistical Analysis in Chemistry and the Chemical Industry*, John Wiley and Sons, New York, 1954, p. 679.
2. Hart, B. I., "Tabulation of the Probabilities for the Ratio of the Mean Square Successive Difference to the Variance," *Annals of Mathematical Statistics*, Vol. 13, 1942, pp. 207–214.
3. Woods, J. D. and Posten, H. O., "Fourier Series and Chebyshev Polynomials in Statistical Distribution Theory," Department of Statistics, University of Connecticut, Research Report No. 37, 1968.

TABLE 1. Lower Critical Values for the Mean Square Successive Difference Test. Upper Critical Value Equals 4 – (Lower Critical Value)

	SIGNIFICANCE LEVEL				SIGNIFICANCE LEVEL				SIGNIFICANCE LEVEL		
N	0.10	0.05	0.01	N	0.10	0.05	0.01	N	0.10	0.05	0.01
10	1.251	1.062	0.752	30	1.543	1.418	1.195	100	1.745	1.674	1.542
11	1.280	1.096	0.792	32	1.557	1.436	1.218	110	1.757	1.689	1.563
12	1.306	1.128	0.828	34	1.569	1.451	1.239	120	1.767	1.702	1.581
13	1.329	1.156	0.862	36	1.581	1.466	1.259	130	1.776	1.714	1.597
14	1.351	1.182	0.893	38	1.592	1.480	1.277	140	1.784	1.724	1.611
15	1.370	1.205	0.922	40	1.602	1.492	1.293	150	1.792	1.733	1.624
16	1.388	1.227	0.949	42	1.611	1.504	1.309	160	1.798	1.741	1.636
17	1.405	1.247	0.974	44	1.620	1.515	1.324	170	1.804	1.749	1.647
18	1.420	1.266	0.998	46	1.628	1.525	1.338	180	1.810	1.756	1.656
19	1.434	1.283	1.020	48	1.635	1.534	1.351	190	1.815	1.763	1.665
20	1.447	1.300	1.041	50	1.642	1.544	1.363	200	1.819	1.768	1.674
21	1.460	1.315	1.060	55	1.659	1.564	1.391	250	1.838	1.793	1.708
22	1.471	1.329	1.078	60	1.673	1.582	1.415	300	1.852	1.811	1.733
23	1.482	1.342	1.096	65	1.685	1.598	1.437	350	1.863	1.825	1.752
24	1.492	1.355	1.112	70	1.697	1.612	1.457	400	1.872	1.836	1.768
25	1.502	1.367	1.128	75	1.707	1.625	1.474	450	1.879	1.845	1.781
26	1.511	1.378	1.143	80	1.716	1.636	1.490	500	1.886	1.853	1.793
27	1.520	1.389	1.157	85	1.724	1.647	1.505	600	1.895	1.866	1.811
28	1.528	1.399	1.170	90	1.732	1.657	1.518	800	1.909	1.884	1.836
29	1.535	1.409	1.183	95	1.739	1.666	1.531	1000	1.919	1.896	1.853