Specification Limits, Capability Indices, and Process Centering in Assembly Manufacture

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The problems with using C_{pk} indices for components to control the capability of an assembly process are well known. Suppliers of components may, for example, actually decrease the yield of conforming product at assembly by raising their own C_{pk}'s. We present formula relationships which link component capability indices (C_p, C_{pm}, C_{pk}) and specifications at the supplier level with capability indices and specifications at the assembly level. The minimum level of C_{pk} (representing a bound on yield) and the minimum C_{pm} (representing centering on target) at the assembly level are derived.

Introduction

MANUFACTURING processes produce characteristics that are often measured in order to determine that these processes conform to *process specifications.* We will consider the common case in which lower and upper specification limits (LSL, USL) are evenly spaced on either side of the target value and the measured characteristics have a normal distribution with mean, μ, and standard deviation, σ. A measurement is *conforming* if it falls within the specification limits. There are various *capability indices* that are used to describe the performance of a process relative to the specification limits. The two most common ones are C_p and C_{pk}, although there is increasing support for C_{pm}, which is now usually included in statistical quality control software. The first two indices are defined as follows:

$$C_p = \frac{USL - LSL}{6\sigma} \tag{1}$$

$$C_{pk} = \min\left(\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right).$$

We will consider the common case in which the target value, τ, is given by

$$\tau = \frac{LSL + USL}{2}. \tag{2}$$

The assumption of a centered target in Equation (2) is the most common case in practice and is the assumption in the preceding work of Boyles (1991). When the target is not in the middle of the specification limits, various anomalies occur in the relationships between indices (Kushler and Hurley (1992)). This situation would be particularly undesirable in the type of application (assembly) we are discussing because prediction of assembly performance from component indices would become impractical. In this case, it can be shown that

$$C_{pk} = \left(1 - \frac{2\,|\delta|}{USL - LSL}\right) C_p = \frac{\frac{1}{2}(USL - LSL) - |\delta|}{3\sigma}, \tag{3}$$

where $\delta = \mu - \tau$ is the "off-centering" of the process.

Basically, C_p compares the 6σ spread of the process (which in the normal distribution contains 99.73% of measurements) to the tolerance spread (specification range). For example, $C_p = 1$ means that if the measurements are from a normal distribution and if the process can be centered at the target, then $\delta = 0$ and 99.73% of the measurements will be conforming. Thus C_p is thought of as the *potential capability* of the process. The related quantity C_{pk} is an attempt to incorporate the off-centering of the process with respect to its effect on the proportion of conforming product. As is seen in Equation (3), the presence of a nonzero δ makes the C_{pk} lower than the C_p.

It can be shown (Boyles (1991)) that the lower bound, Y_{LB}, on the proportion of product conform-

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ing (yield), Y_d, is

$$Y_{LB} = 2\Phi(3C_{pk}) - 1, \tag{4}$$

where Φ is the standardized normal cumulative distribution. When the process is centered and $\delta = 0$, one obtains $C_p = C_{pk}$ and $Y_d = Y_{LB}$.

An alternative approach to incorporating the effects of off-centering is the C_{pm} capability index, where

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + \delta^2}}, \tag{5}$$

which can be interpreted in terms of the Taguchi loss function (Johnson (1992)). This index is also discussed in Boyles (1991) and Spiring (1991). The effect of δ can easily be seen by comparing Equations (1) and (5).

Given our assumption in Equation (2), the following interrelationships can be derived:

$$C_{pk} = C_p - \frac{1}{3}\sqrt{\left(\frac{C_p}{C_{pm}}\right)^2 - 1}, \tag{6}$$

$$C_{pm} = \frac{C_p}{\sqrt{1 + 9(C_p - C_{pk})^2}}. \tag{7}$$

Note that $C_p \geq C_{pm}$ and $C_p \geq C_{pk}$.

The relationships in Equations (6) and (7) are illustrated in Figures 1 and 2, which provide valuable insights into the nature of these indices. For example, in Figure 1 it is seen that $C_p = 2.8$ and $C_{pk} = 1.8$, both "high" values of these indices, give a C_{pm} of only 0.9. This is because C_p and C_{pk} are essentially concerned with the yield of a process, not centering on target. A C_p value substantially higher than the C_{pk} indicates that the process is considerably off-center from the mid-point of the specifications.

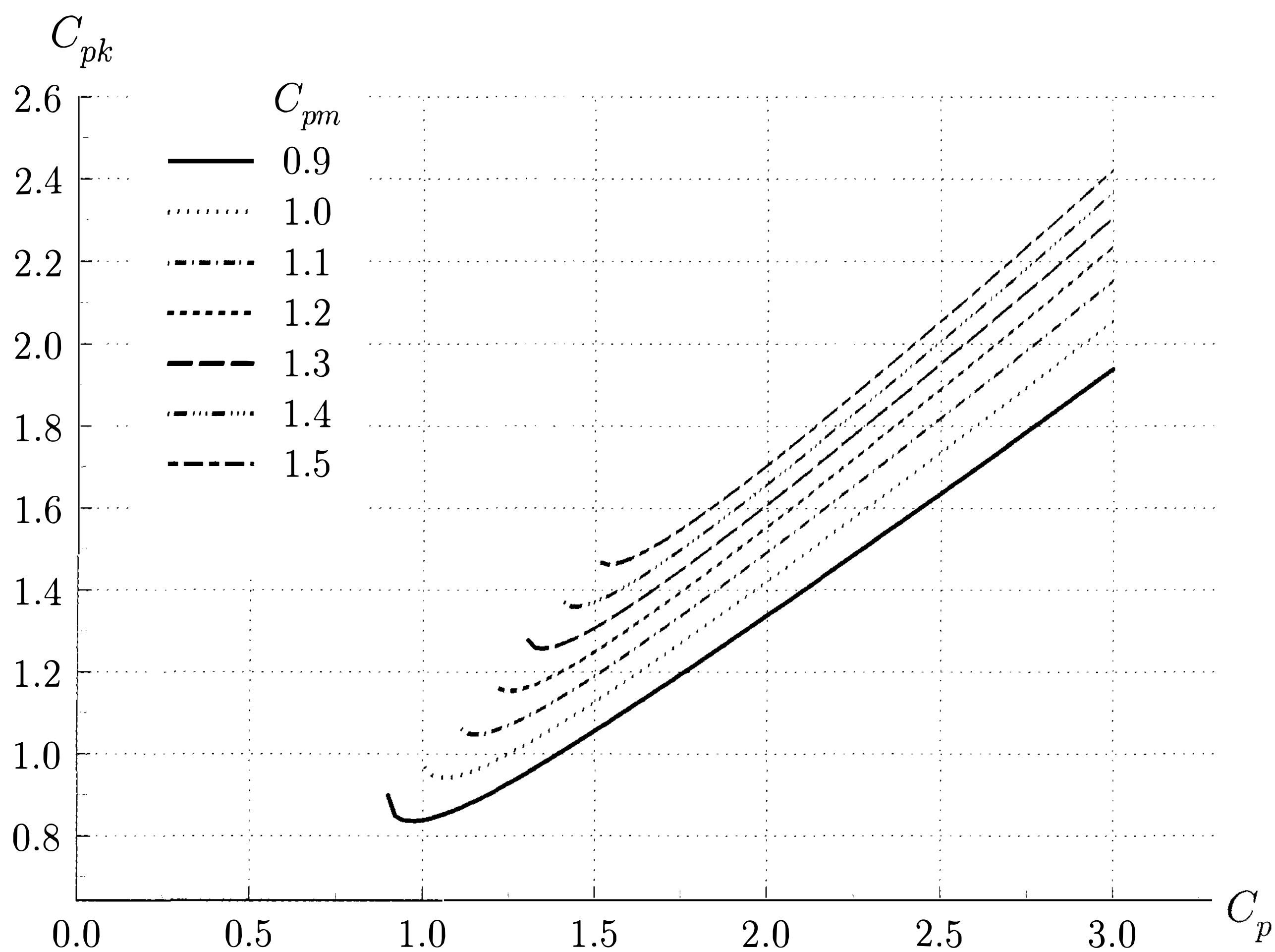

FIGURE 1. C_{pk} as a Function of C_p for Given C_{pm}.

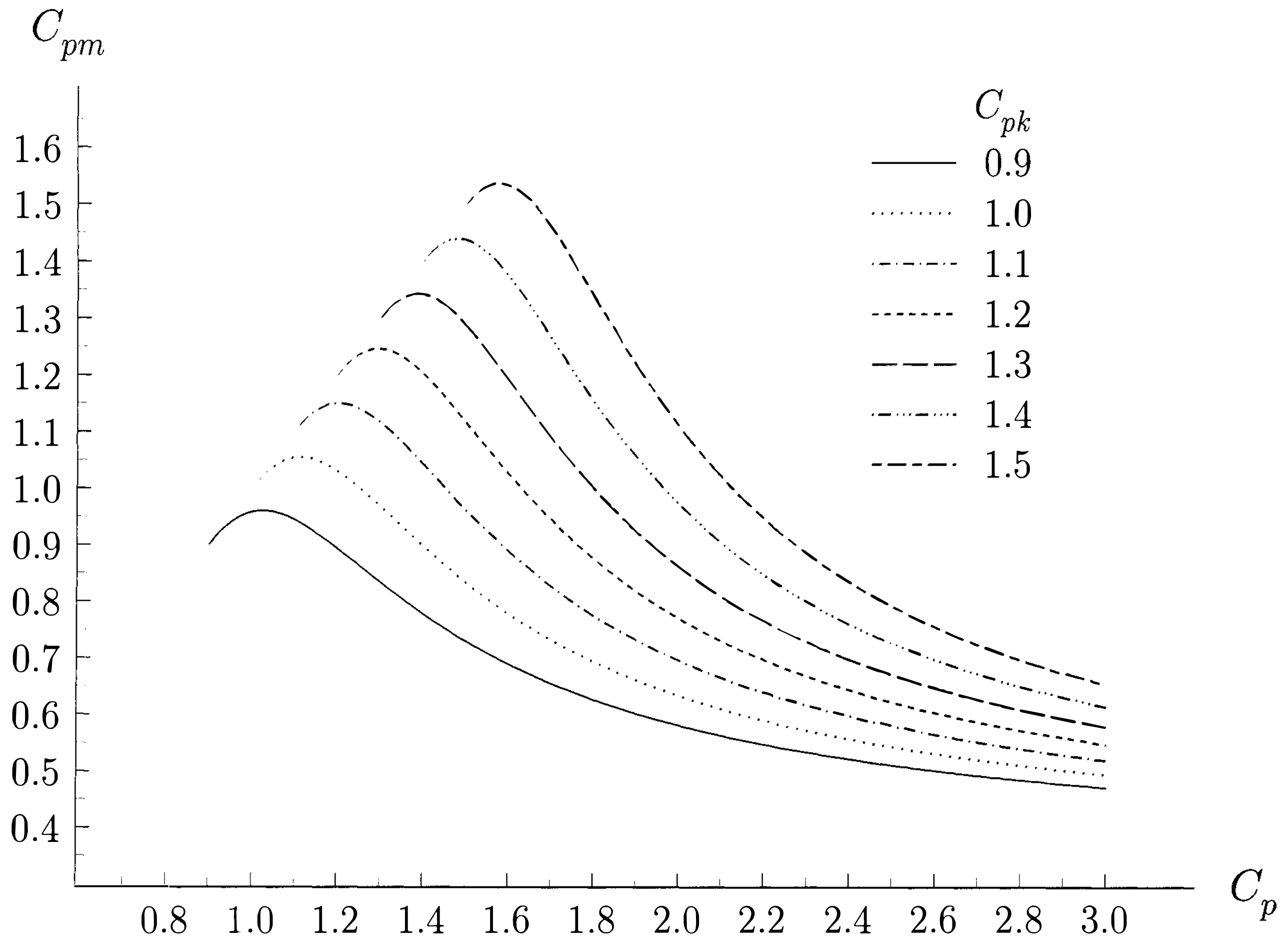

FIGURE 2. C_{pm} as a Function of C_p for Given C_{pk}.

This point is made in a slightly different way in Figure 2. For a given C_{pk}, C_{pm} reaches its maximum at $C_p = (9C_{pk}^2 + 1)/(9C_{pk})$. Almost invariably, if C_p is raised while holding C_{pk} constant, the result is a drop in C_{pm}. As we shall see, this could have unfortunate effects on the yield of the assembly process.

Although indices such as these are being increasingly used (Barnett (1990) and McCoy (1991)), there is considerable concern that they may also be misused (Nelson (1992) and Gunter (1989)). One major concern is the robustness of the normality assumption (Bates and English (1991)). Another concern is that practitioners may not adequately take into account the variability of the sample statistics by which these indices are estimated (Pearn, Kotz, and Johnson (1992) and Kushler and Hurley (1992)). Overviews are given in Rodriguez (1992) and Porter and Oakland (1991).

A third concern is that the C_{pk} index may be dangerously inadequate in assuring process centering in components of a more complex assembly (Boyles (1991)). It is this problem that this paper attempts to address. Although we deal with several processes at once, we are not using multivariate process capability indices in the sense described, for example, in Chapter 5 of Kotz and Johnson (1993). We link the univariate capability indices measured at the component processes with the capability indices at the process where these components undergo *random assembly* (DeVor, Chang, and Sutherland (1992, page 269)). The processes are thus uncorrelated.

This paper does not deal with two issues of concern in the literature on indices cited above—the effects of non-normality and the problems of estimation of indices from samples. These issues are not within the scope of this paper. However, a primary purpose of our analysis is cautionary, and these complications would urge even more caution.

In the next section, we establish notation for an assembly process which may be, at least approximately, described by a linear function of n component processes. Then a simple numerical example illustrates the notation and shows how setting standards and capability indices for suppliers (components) may lead to unfortunate consequences on the yield of the assembly process.

We then develop explicit formula relationships linking process specifications and capability indices for component suppliers to either a minimum level of C_{pk} or a minimum level of C_{pm} at the assembly process (assuming random assembly). In the first instance, the goal is a minimum level of yield at assembly; in the second instance, the goal is a well-centered process which, in turn, may be a component for a higher level of assembly. The appropriate goal in any given instance could be a matter of controversy. The analysis uses C_{pm} indices for suppliers. It is possible to use a combination of C_p and C_{pk} indices for this purpose (using Equation (6) to find an equivalent combination), but, as will be discussed briefly, this could be somewhat risky. We present numerical examples for the $n = 2$ and $n = 3$ cases.

A Formulation for a Multi-Component Assembly

Let

$$X_0 = a_0 + \sum_{i=1}^{n} a_i X_i \qquad (8)$$

be an assembly function, where X_0 is the measurement on the final assembly and X_i is the measurement on component i for $i = 1, \ldots, n$. Actually, X_0 could be a nonlinear function of X_i's for $i > 0$; then Equation (8) could be a linear expression derived from a first-order Taylor approximation (Boyles (1991)). Without loss of generality for our analysis, we can assume that $a_0 = 0$.

We assume that each X_i has a normal distribution and that the target value for each process is the center point of its specifications. Let LSL_i and USL_i be, respectively, the lower and upper specification limits on X_i, and let the range be $\mathcal{R}_i = USL_i - LSL_i$. Denote the mean and standard deviations of X_i by μ_i and σ_i, respectively, and let

$$\tau_i = \frac{1}{2}(USL_i + LSL_i),$$

where $\delta_i = \mu_i - \tau_i$ is the off-centering of component i. It follows that

$$\mu_0 = \sum_{i=1}^{n} a_i \mu_i$$
$$\sigma_0 = \sqrt{\sum_{i=1}^{n} a_i^2 \sigma_i^2} \qquad (9)$$
$$\delta_0 = \sum_{i=1}^{n} a_i \delta_i .$$

The C_{pk} coefficient at component i is given by

$$C_{pk}^{(i)} = \left(1 - \frac{|\delta_i|}{\mathcal{R}_i/2}\right) C_p^{(i)} = \frac{\mathcal{R}_i/2 - |\delta_i|}{3\sigma_i},$$

where

$$C_p^{(i)} = \frac{\mathcal{R}_i}{6\sigma_i}.$$

A lower bound on $Y_d^{(0)}$, the yield of assembled product, can be obtained from the $C_{pk}^{(0)}$ index by employing Equation (4).

Controlling Components With C_{pk}: An Example

As has been pointed out by Boyles (1991) and others, specifying only C_{pk} for suppliers may lead to difficulties. Consider a simple hypothetical example in which two components are joined together (e.g., two washers used as a spacer). Hence, $n = 2$, $a_1 = 1$, $a_2 = 1$, and, by Equation (8), $X_0 = X_1 + X_2$. Assume $\sigma_1 = \sigma_2 = 0.001$, $\mathcal{R}_1 = \mathcal{R}_2 = 0.006$, and $\mathcal{R}_0 = 0.008485$. Also assume that initially both suppliers of washers have their processes centered between the specification limits (i.e., $\delta_1 = \delta_2 = 0$). Then

$$\sigma_0 = \sqrt{0.001^2 + 0.001^2} = 0.0014142,$$

and, substituting into Equations (1) and (3),

$$C_{pk}^{(0)} = 1.00, \quad C_{pk}^{(1)} = 1.00, \quad C_{pk}^{(2)} = 1.00.$$

Using Equation (4) to calculate the yield at the assembly stage and at each component stage, we obtain

$$Y_d^{(i)} = 2\Phi[3C_{pk}^{(i)}] - 1 = 0.9973 \quad \text{for } i = 0,\ 1,\ 2\,.$$

Now suppose that the supplier of Component 1 discovers a method to reduce the process standard deviation of Component 1 to 0.0001, giving $C_p = 10$. The required $C_{pk}^{(1)} = 1$ can now be met with a lower μ_1, resulting in a savings in material. Solving $1 = (1 - |\delta_1|/0.003)10$, this supplier gets $\delta_1 = \pm 0.0027$ and so reduces the thickness of the washer by 0.0027.

However, calculating the C_{pk} for the assembly stage and using Equation (3), we find

$$C_{pk}^{(0)} = \left(1 - \frac{0.0027}{0.008485/2}\right)\frac{0.008485}{6\sqrt{0.0001^2 + 0.001^2}} = 0.51\,,$$

which would not generally be considered an acceptable C_{pk} for a process. This example illustrates the pitfalls of trying to control suppliers with C_{pk} when dealing with an assembled product.

Predicting Assembly C_{pk} from Component C_{pm}

An alternative to C_{pk} that has been recommended for situations such as the one described above is requiring component processes to maintain a specified C_{pm} (in addition to specified specification limits), where

$$C_{pm}^{(i)} = \frac{\mathcal{R}_i}{6\sqrt{\sigma_i^2 + \delta_i^2}} \qquad \text{for } i = 1, \ldots, n.$$

We now formulate $C_{pk}^{(0)}$ in terms of $\mathcal{R}_i$ and $C_{pm}^{(i)}$ for $i = 1, \ldots, n$ in order to establish a lower limit on $C_{pk}^{(0)}$ in terms of the $C_{pm}^{(i)}$, which are the specification limits for the suppliers.

From Equation (5), we obtain, for each component process,

$$\sigma_i^2 = \frac{\mathcal{R}_i^2 - [6C_{pm}^{(i)}\delta_i]^2}{[6C_{pm}^{(i)}]^2} \qquad \text{for } |\delta_i| \le \frac{\mathcal{R}_i}{6C_{pm}^{(i)}}. \tag{10}$$

Note that if $|\delta_i| > \mathcal{R}_i/[6C_{pm}^{(i)}]$, the process could not meet the C_{pm} requirement—even with $\sigma_i = 0$.

Substituting Equation (10) into Equation (3), we obtain

$$C_{pk}^{(0)} = \frac{\mathcal{R}_0/2 - \left|\sum_{i=1}^{n} a_i\delta_i\right|}{3\sqrt{\sum_{i=1}^{n} a_i^2\left(\frac{\mathcal{R}_i^2 - [6C_{pm}^{(i)}\delta_i]^2}{[6C_{pm}^{(i)}]^2}\right)}}. \tag{11}$$

This is the C_{pk} at the assembly stage. Its value depends on the δ_i's at the component suppliers. For example, if the δ_i's can be made zero, then the relationships are relatively simple; that is, $C_p^{(i)} = C_{pk}^{(i)} = C_{pm}^{(i)}$. Then

$$C_p^{(0)} = \frac{\mathcal{R}_0}{\sqrt{\sum_{i=1}^{n} \frac{a_i^2\mathcal{R}_i^2}{[C_p^{(i)}]^2}}},$$

and

$$\frac{\mathcal{R}_i}{C_p^{(i)}} = \frac{1}{|a_i|}\sqrt{\frac{\mathcal{R}_0^2}{[C_p^{(0)}]^2} - \sum_{\substack{j=1\\ j\ne i}}^{n} \frac{a_j^2 R_j^2}{[C_p^{(j)}]^2}}.$$

The latter relationship means that if the specification limits $\mathcal{R}_i$ of $n-1$ components are given, the remaining one which is required to produce the specified $C_p^{(0)}$ can be found.

The question we ask is: What is the consequence on the assembly process if the suppliers "do their worst"? This is not a suggestion that they should; it is an attempt to find a lower bound on the yield at assembly.

Let

$$C_{\min} = \min_{\delta_1,\ldots,\delta_n} C_{pk}^{(0)}.$$

Direct minimization using extreme conditions yields

Result 1:

$$\delta_i^* = \frac{1}{18}\,\frac{\sum_{j=1}^{n} a_j^2\mathcal{R}_j^2\left(\prod_{\substack{\ell=1\\ \ell\ne j}}^{n}[C_{pm}^{(\ell)}]^2\right)}{\mathcal{R}_0 a_i \prod_{j=1}^{n}[C_{pm}^{(j)}]^2}. \tag{12}$$

Note that, as can be seen from Equation (11), if $\Delta^* = (\delta_1^*, \ldots, \delta_n^*)$, then $-\Delta^*$ is also a solution.

Now, substituting Equation (12) into Equation (11) gives

Result 2:

$$C_{\min} = \frac{\sqrt{9\mathcal{R}_0^2 \prod_{i=1}^{n}[C_{pm}^{(i)}]^2 - n\sum_{i=1}^{n} a_i^2\mathcal{R}_i^2\left(\prod_{\substack{j=1\\ j\ne i}}^{n}[C_{pm}^{(j)}]^2\right)}}{3\sqrt{\sum_{i=1}^{n}\left[a_i^2\mathcal{R}_i^2\left(\prod_{\substack{j=1\\ j\ne i}}^{n}[C_{pm}^{(j)}]^2\right)\right]}}. \tag{13}$$

The relationship in Equation (13) can be used to set specification limits and C_{pm}'s for component suppliers. Rearranging, we have

Result 3:

$$\frac{\mathcal{R}_i}{C_{pm}^{(i)}} = \frac{\sqrt{\frac{9\mathcal{R}_0^2\prod_{\substack{j=1\\ j\ne i}}^{n}[C_{pm}^{(j)}]^2}{(n+9C_{\min}^2)} - \sum_{\substack{j=1\\ j\ne i}}^{n} a_j^2\mathcal{R}_j^2\left(\prod_{\substack{t=1\\ t\ne j,\, t\ne i}}^{n}[C_{pm}^{(t)}]^2\right)}}{|a_i|\prod_{\substack{j=1\\ j\ne i}}^{n} C_{pm}^{(j)}}. \tag{14}$$

This means that if the assembly process is given a value for minimum acceptable C_{pk} (i.e., $C_{\min}$), then Equation (14) enables one to set the specifications *versus* C_{pm} tradeoffs for component process j, provided the specification limits and C_{pm}'s for the other components are known.

It should be noted that Result 1 provides an *unconstrained* minimum. In other words, it is possible that δ_i^* violates the constraint $|\delta_i| \le \mathcal{R}_i/[6C_{pm}^{(i)}]$ in Equation (10). This generally occurs if a_i is very small. However, an unconstrained minimum is always as small or smaller than a constrained minimum. Therefore, if we set requirements according to $C_{\min}$, we are guaranteed a value of C_{pk} at assembly at least as large as this. The quantity $C_{\min}$ is therefore a lower bound on the minimum of $C_{pk}^{(0)}$. Incidentally, we can check whether for every i, $|\delta_i^*| \le \mathcal{R}_i/[6C_{pm}^{(i)}]$; if this is true, then our bound is "tight."

A special case is when the C_{pm}'s and $\mathcal{R}$'s for each component process are to be the same, in which case we obtain

$$\frac{\mathcal{R}_i}{C_{pm}^{(i)}} = \frac{3\mathcal{R}_0}{\sqrt{\sum_{j=1}^{n} a_j^2 \sqrt{n + 9C_{\min}^2}}}, \quad i = 1, \ldots, n. \tag{15}$$

Predicting Assembly C_{pm} from Component C_{pm}

Instead of designing specification limits and C_{pm} requirements for the components to assure a minimum level of $C_{pk}^{(0)}$ (C_{pk} at assembly), we could set the assembly requirement in terms of $C_{pm}^{(0)}$. Controlling assembly centering instead of yield is a more "Taguchi-correct" concept. Also, the assembly process could itself be a component for a following process.

Substituting Equation (10) into Equation (5), we obtain

$$C_{pm}^{(0)} = \frac{\mathcal{R}_0}{6\sqrt{\sum_{i=1}^{n} a_i^2 \left(\frac{\mathcal{R}_i^2 - [6C_{pm}^{(i)}\delta_i]^2}{[6C_{pm}^{(i)}]^2}\right) + \left(\sum_{i=1}^{n} a_i \delta_i\right)^2}}. \tag{16}$$

The value of the C_{pm} at the assembly stage also depends on the δ_i's at the component suppliers. We again investigate the worst possible consequences on the assembly process.

Let

$$\zeta_{\min} = \min_{\delta_1, \ldots, \delta_n} C_{pm}^{(0)}.$$

Analysis of the derivatives of $C_{pm}^{(0)}$ with respect to the δ_i's shows that the minimum is achieved in the somewhat improbable case in which the component processes have all been reduced to zero variance and thus have maximum off-centering. In other words, the constraints in Equation (10) give the magnitude of the δ_i's.

Result 4:

The minimum value $\zeta_{\min}$ occurs when

$$\tilde{\delta}_i = \text{Sign}(a_i)\frac{\mathcal{R}_i}{6C_{pm}^{(i)}}.$$

Substituting this result into Equation (16) gives

Result 5:

$$\zeta_{\min} = \frac{\mathcal{R}_0 \prod_{i=1}^{n} C_{pm}^{(i)}}{\sqrt{RC + \sum_{i=1}^{n} a_i^2 \mathcal{R}_i^2 \prod_{\substack{j=1 \\ j \ne i}}^{n} [C_{pm}^{(j)}]^2}},$$

where

$$RC = 2\sum_{i=1}^{n} \sum_{j=i+1}^{n} |a_i b_j| \mathcal{R}_i \mathcal{R}_j C_{pm}^{(i)} C_{pm}^{(j)} \prod_{\substack{t=1 \\ t \ne i,\, t \ne j}}^{n} [C_{pm}^{(t)}]^2.$$

We can again state the requirements on components in terms of $\zeta_{\min}$ and the requirements on the other suppliers as

Result 6:

$$\frac{\mathcal{R}_i}{C_{pm}^{(i)}} = \frac{1}{|a_i|}\left(\frac{\mathcal{R}_0}{\zeta_{\min}} - \sum_{\substack{j=1 \\ j \ne i}}^{n} \frac{\mathcal{R}_j |a_j|}{C_{pm}^{(j)}}\right).$$

Like Result 3, Result 6 can be used to give requirements to the component processes, but this time to control the C_{pm} at the assembly process.

Example

The following hypothetical and simplified example illustrates the preceding analysis. A milled slot has two metal inserts, as shown in cross-section in Figure 3. The specification limits for the slot width are currently 3.0 ± 0.006 inches. The specification limits for the inserts, which are of different materials, are 1.1 ± 0.003 and 1.8 ± 0.004 inches. The C_p for both

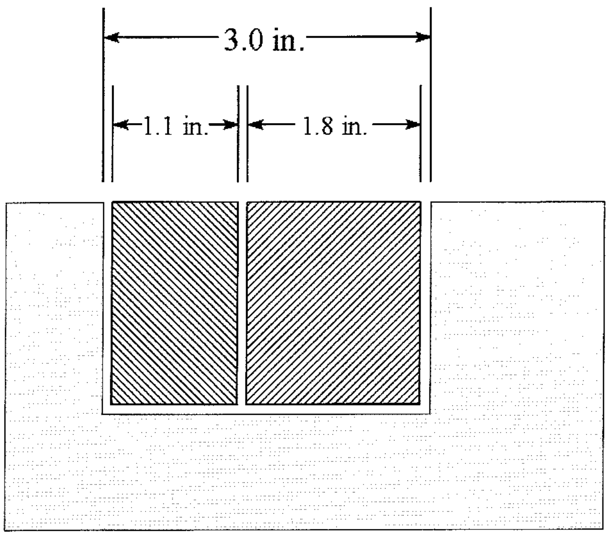

FIGURE 3. A Slot and Two Inserts.

the slot and the inserts are to be 1.1. The specifications for the clearance (the difference between slot width and combined insert widths) is 0.1 ± 0.008 inches. Normality is assumed for all processes.

Let X_0 be the clearance, X_1 be the slot width, X_2 be the width of Insert 1, and X_3 be the width of Insert 2. Thus, $X_0 = X_1 - X_2 - X_3$, $a_1 = 1$, $a_2 = -1$, and $a_3 = -1$. By our notation, $\mathcal{R}_0 = 0.016$, $\mathcal{R}_1 = 0.012$, $\mathcal{R}_2 = 0.006$, and $\mathcal{R}_3 = 0.008$.

Since $C_p = 1.1$ for all three processes, it follows from Equation (1) that $\sigma_1 = 0.0018182$, $\sigma_2 = 0.0009091$, $\sigma_3 = 0.001212$, and hence, by Equation (9),

$$\sigma_0 = \text{sqrt}\Big\{1^2(0.0018182)^2 + (-1)^2(0.0009091)^2 + (-1)^2(0.001212)^2\Big\} = 0.00237\,.$$

If all three component processes are centered, then $\delta_0 = 0$ and $C_{pk}^{(0)} = 0.016/[6(0.00237)] = 1.13$.

Suppose now that the only requirement on the three component processes is that they maintain $C_{pm} = 1.1$. That is, they are free to off-center their processes provided that the process variance is decreased sufficiently so that this C_{pm} is maintained. We can use Equation (13) to calculate the "worst possible" C_{pk} for clearance.

For the case where $n = 3$, $C_{\min}$ of Equation (13) becomes

$$\text{sqrt}\Big\{9\mathcal{R}_0^2[C_{pm}^{(1)}]^2[C_{pm}^{(2)}]^2[C_{pm}^{(3)}]^2 - 3\{a_1^2\mathcal{R}_1^2[C_{pm}^{(2)}]^2[C_{pm}^{(3)}]^2 + a_2^2\mathcal{R}_2^2[C_{pm}^{(1)}]^2[C_{pm}^{(3)}]^2 + a_3^2\mathcal{R}_3^2[C_{pm}^{(1)}]^2[C_{pm}^{(2)}]^2\}\Big\} \times \Big[3\ \text{sqrt}\Big\{a_1^2\mathcal{R}_1^2[C_{pm}^{(2)}]^2[C_{pm}^{(3)}]^2 + a_2^2\mathcal{R}_2^2[C_{pm}^{(1)}]^2[C_{pm}^{(3)}]^2 + a_3^2\mathcal{R}_3^2[C_{pm}^{(1)}]^2[C_{pm}^{(2)}]^2\Big\}\Big]^{-1}.$$

Substituting our parameters, we obtain

$$C_{\min} = \text{sqrt}\Big\{9(0.016)^2(1.1)^6 - 3[1^2(0.012)^2(1.1)^4 + (-1)^2(0.006)^2(1.1)^4 + (-1)^2(0.008)^2(1.1)^4]\Big\} \times \Big[3\ \text{sqrt}\Big\{1^2(0.012)^2(1.1)^4 + (-1)^2(0.006)^2(1.1)^4 + (-1)^2(0.008)^2(1.1)^4\Big\}\Big]^{-1} = 0.9675\,.$$

To check if our bound is tight, we can calculate that $|\tilde{\delta}_i| = 0.0007002$ for all i. Also,

$$\frac{\mathcal{R}_1}{6C_{pm}^{(1)}} = 0.0018, \quad \frac{\mathcal{R}_2}{6C_{pm}^{(2)}} = 0.000909,$$

$$\text{and} \quad \frac{\mathcal{R}_3}{6C_{pm}^{(3)}} = 0.001212.$$

Therefore, in this example, $C_{\min}$ is a tight, or possible, lower bound.

In order to ensure that the C_{pk} at assembly is at least 1.1, the specification limits and/or the C_{pm} at the components must be reconsidered. Let us assume that the variance in the slot milling process can be improved. Applying Equation (14),

$$\frac{\mathcal{R}_1}{C_{pm}^{(1)}} = \frac{1}{a_1 C_{pm}^{(2)} C_{pm}^{(3)}} \times \text{sqrt}\Big\{\frac{9\mathcal{R}_0^2[C_{pm}^{(2)}]^2[C_{pm}^{(3)}]^2}{3 + 9C_{\min}^2} - a_2^2\mathcal{R}_2^2[C_{pm}^{(3)}]^2 - a_3^2\mathcal{R}_3^2[C_{pm}^{(2)}]^2\Big\}$$

$$= \frac{1}{1(1.1)^2} \times \text{sqrt}\left\{ \frac{9(0.016)^2(1.1)^4}{3+9(1.1)^2} - (-1)^2(0.006)^2(1.1)^2 - (-1)^2(0.008)^2(1.1)^2 \right\}$$
$$= 0.009123,$$

and hence the new width of the specification limits $\mathcal{R}_1$ should be $(0.009123)(1.1) = 0.0100$.

If this were a case where the specification limits for all three component processes were to be changed and made equal, then, by Equation (15),

$$\frac{\mathcal{R}_1}{C_{pm}^{(1)}} = \frac{3\mathcal{R}_0}{\sqrt{\sum_{i=1}^{n} a_i^2}\sqrt{n+9C_{\min}^2}} = \frac{3(0.016)}{\sqrt{3}\sqrt{3+9(1.1)^2}} = 0.007436$$

and $\mathcal{R}_1 = \mathcal{R}_2 = \mathcal{R}_3 = 0.00818$.

We now turn to our alternative criterion at assembly, namely, C_{pm}. If all three component processes are centered, then $C_{pm} = 1.267$, which is the same as C_{pk}. We also have

$$\zeta_{\min} = \frac{\mathcal{R}_0 C_{pm}^{(1)} C_{pm}^{(2)} C_{pm}^{(3)}}{\sqrt{T_1+T_2}},$$

where

$$T_1 = 2\,|a_1a_2|\,\mathcal{R}_1\mathcal{R}_2 C_{pm}^{(1)} C_{pm}^{(2)} [C_{pm}^{(3)}]^2 + 2\,|a_1a_3|\,\mathcal{R}_1\mathcal{R}_3 C_{pm}^{(1)} [C_{pm}^{(2)}]^2 C_{pm}^{(3)} + 2\,|a_2a_3|\,\mathcal{R}_2\mathcal{R}_3 [C_{pm}^{(1)}]^2 C_{pm}^{(2)} C_{pm}^{(3)}$$
$$T_2 = a_1^2\mathcal{R}_1^2[C_{pm}^{(2)}C_{pm}^{(3)}]^2 + a_2^2\mathcal{R}_2^2[C_{pm}^{(1)}C_{pm}^{(3)}]^2 + a_3^2\mathcal{R}_3^2[C_{pm}^{(1)}C_{pm}^{(2)}]^2.$$

Substituting, we obtain $\zeta_{\min} = 0.677$, which is considerably below the "centered" value of 1.267.

Suppose that the requirement for $\zeta_{\min}$ is 1.1. We can use Result 6 to investigate what changes to component specifications should be made. For example, consider the following slot process:

$$\frac{\mathcal{R}_1}{C_{pm}^{(1)}} = \frac{1}{|a_1|}\left(\frac{\mathcal{R}_0}{\zeta_{\min}} - \sum_{j=2}^{n}\frac{\mathcal{R}_j\,|a_j|}{C_{pm}^{(j)}}\right) = \frac{1}{1}\left(\frac{0.016}{1.1} - \frac{0.006\,|-1|}{1.1} - \frac{0.008\,|-1|}{1.1}\right) = 0.001818,$$

which gives $\mathcal{R}_1 = 0.002$, a much smaller value than the current one of 0.012. It could be advisable to tighten the requirements at the other components so that the slot requirement would not be so strict.

Concluding Remarks

This paper has given a relatively simple method for determining specification limits and process capability indices for component suppliers in terms of either a desired C_{pk} or C_{pm} at the assembly process. Although the capability indices specified for the component processes are C_{pm}'s, they could be transformed into equivalent C_p and C_{pk} indices using Equation (6). In this case it would be imperative, however, that the C_p indices are the actual indices for the process. If the requirement for a supplier called for a C_p of 1.3 and a C_{pk} of 1.3, for example, then a $C_p = 1.5$ would not be "better" because it could allow off-centering to a degree that would lower the yield or lower the C_{pm} at the assembly process.

Acknowledgments

This research was supported in part by the Natural Sciences and Engineering Research Council of Canada. The authors wish to thank two anonymous referees for their insightful comments on an earlier version of the paper.

References

BARNETT, N. S. (1990). "Process Control and Product Quality". *International Journal of Quality and Reliability Management* 7, pp. 34–43.

BATES, T. E. and ENGLISH, J. R. (1991). "The Robustness of Modern Process Capability Analysis". *Computers and Industrial Engineering* 21, pp. 45–49.

BOYLES, R. A. (1991). "The Taguchi Capability Index". *Journal of Quality Technology* 23, pp. 17–26.

DEVOR, R. E.; CHANG, T.; and SUTHERLAND, J. W. (1992). *Statistical Quality Design and Control*. Macmillan, New York, NY.

GUNTER, B. (1989). "The Use and Abuse of C_{pk}—Part 4". *Quality Progress* 22, pp. 86–87.

JOHNSON, T. (1992). "The Relationship of C_{pm} to Squared Error Loss". *Journal of Quality Technology* 24, pp. 211–215.

KOTZ, S. and JOHNSON, N. L. (1993). *Process Capability Indices*. Chapman and Hall, London, UK.

KUSHLER, R. H. and HURLEY, P. (1992). "Confidence Bounds for Capability Indices". *Journal of Quality Technology* 24, pp. 188–195.

MCCOY, P. F. (1991). "Using Performance Indexes to Monitor Production Processes". *Quality Progress* 24, pp. 49–55.

NELSON, P. R. (1992). Editorial. *Journal of Quality Technology* 24, p. 175.

PEARN, W. L.; KOTZ, S.; and JOHNSON, N. L. (1992). "Distributional and Inferential Properties of Process Capability Indices". *Journal of Quality Technology* 24, pp. 216–231.

PORTER, L. J. and OAKLAND, J. S. (1991). "Process Capability Indices: An Overview of Theory and Practice". *Quality and Reliability Engineering Journal* 7, pp. 437–448.

RODRIGUEZ, R. N. (1992). "Recent Developments in Process Capability Analysis". *Journal of Quality Technology* 24, pp. 176–187.

SPIRING, F. A. (1991). "C_{pm} Index". *Quality Progress* 24, pp. 57–61.

Key Words: *Capability Indices, Statistical Process Control.*